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N. Cabibbo, G. Da Prato, G. De Franceschi, U. Mosco:

CIRCULAR POLARIZATION OF HIGH ENERGY γ- RAYS BY BIREFRANGENCE IN CRYSTALS.

Nota interna: n° 160
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1) In a previous work(1) the authors have shown that thick single crystals can be used as polarizers and polarimeters for high energy gamma rays. This possibility is due to the well known coherence effects in pair production, which cause a dependence of the absorption cross section on the linear polarization of the photons.

In this paper we show that as a consequence of the effects discussed in A and B single crystals are also birefrangent. This opens interesting possibilities in the use of single crystals for the handling of circularly polarized gamma rays. In particular it is possible to use a crystal of appropriate thickness as the high energy analogue of a quarter wave lenght plate to convert linear into circular polarization, and viceversa. In this way it will be possible to produce and analize circularly polarized gamma ray beams of very high energy. As in the case of linear polarization[1] the efficiency is found to increase with the energy.

2) As in A we consider a cubic crystal of thickness x and a photon whose momentum $\vec{K}$ is in the (001) plane of the crystal at an angle δ from the (110) axis. The polarization vector $\vec{\varepsilon}$ of the incoming photon will in general be a combination of two vectors $\vec{F}$ and $\vec{y}$ (see B) respectively on the (001) plane and orthogonal to it: $\vec{\varepsilon} = \varepsilon_1 \vec{F} + \varepsilon_2 \vec{y}$.

$\vec{\varepsilon}$ can be considered as a two component vector : $\vec{\varepsilon} \equiv (\varepsilon_1, \varepsilon_2)$. After the crystal the outgoing amplitude in the forward channel (photon of momentum K) will be connected to $\vec{\varepsilon}$ by a 2 x 2 diagonal matrix of the form[2]

$$S = \begin{pmatrix} e^{in^{\parallel}(\omega,\delta)\omega x} & 0 \\ 0 & e^{in^{\perp}(\omega x)\omega x} \end{pmatrix}$$

where $\omega = |\vec{K}|$[3] and the quantities $n^{\parallel}$ and $n^{\perp}$ are the analogues of the refraction index in optics.

The crystal acts as a 1/4 λ plate if the relative phase of the two components is changed by $\frac{\pi}{2}$ i.e. if $Re(n^{\perp} - n^{\parallel})\omega x = \frac{\pi}{2}$.

The imaginary parts of these are connected with the absorption cross section: $Im\, n(\omega) = \frac{\Sigma(\omega)}{2\omega}$ and were discussed and evaluated in A and B. The real parts can be derived from them by the use of dispersion, relations[4]. We are interested in the difference of the real parts, which enters in the phase relations between the two components:

1) $$Re(n^{\perp}(\omega,\delta) - n^{\parallel}(\omega,\delta)) = \frac{1}{\pi} P \int_0^{\infty} \frac{\Sigma^{\perp}(\omega',\delta) - \Sigma^{\parallel}(\omega',\delta)}{\omega'^2 - \omega^2} d\omega'$$

From B one has[5]:

2) $$\Sigma^{\perp}(\omega,\delta)-\Sigma^{\parallel}(\omega,\delta) = \sum_{q} K(q)\frac{1}{\omega\beta^2}\left(\beta\sqrt{1-\frac{4}{\beta}} + 2\ln\frac{1+\sqrt{1-\frac{4}{\beta}}}{1-\sqrt{1-\frac{4}{\beta}}}\right)\theta(\beta-4)$$

with

3) $$K(\underline{q}) = \frac{8\alpha Z^2 b^2}{\pi}\,\frac{N}{a^3}\,(2\pi)^3\,\frac{b^4 e^{-Aq^2}}{(1+b^2q^2)^2}\cos 2\theta\, D(\underline{q})\,\frac{4q^2\omega^2-\beta^2}{\beta^2}$$

where b is the screening lenght, N the number of atoms per unit volume, a the lattice constant, $D(\underline{q})$ and A are defined in B; $\beta = 2(\underline{k}.\underline{q})$, θ is the angle between the (100) and the $\underline{k}$, $\underline{q}$ planes. Integrating term by term, we obtain:

4) $$\mathrm{Re}(n^{\perp}-n^{\parallel}) = \sum_{\alpha} f(\omega,\delta,\beta)$$

and

5) $$f(\omega,\delta,\beta) = \frac{1}{8\pi}\frac{1}{\omega^2}K(\underline{q})\left\{\left[\sqrt{1-4/\beta} + \frac{2}{\beta}\ln\frac{1+\sqrt{1-4/\beta}}{1-\sqrt{1-4/\beta}}\right]^2 - \frac{4\pi^2}{\beta^2} + \left[\sqrt{1+4/\beta} - \frac{2}{\beta}\ln\frac{\sqrt{1+4/\beta}+1}{\sqrt{1+4/\beta}-1}\right]^2\right\} \quad \text{for } \beta > 4$$

$$f(\omega,\delta,\beta) = \frac{1}{8\pi}\frac{1}{\omega^2}K(\underline{q})\left\{-\left[\sqrt{4/\beta-1} - \frac{4}{\beta}\cot^{-1}\sqrt{4/\beta-1}\right]^2 + \left[\sqrt{1+4/\beta} - \frac{2}{\beta}\ln\frac{\sqrt{1+4/\beta}+1}{\sqrt{1+4/\beta}-1}\right]^2\right\} \quad \text{for } \beta \leq 4$$

The summation is extended on the points in the reciprocal lattice for which $(\underline{k}.\underline{q}) > 0$.

3) Numerical evaluation of equation 4) for the case of a Cu crystal have been done with an IBM 1620 computer. In table I we give the "best results" around 1 GeV, 6 GeV, 40 GeV, and

the thickness of a $\lambda/4$ plate, $x = \frac{\pi}{2\omega}(n^{\perp} - n^{\parallel})$.

TABLE I

ω GeV	δ mrad	$Re(n^{\perp} - n^{\parallel})$	X in cm
1	28	2.62	11.5
6	3.7	2.74	1.84
40	0.46	2.67	0.273

It is seen that the effect at the best angle is nearly independent from the energy. In fig. 1 and 2 we give the dependence of $Re(n^{\perp} - n^{\parallel})$ on the angle and the energy in the 6 GeV region. For comparison these figures contain also a plot of $E(\delta, \omega) = (\Sigma^{\perp} - \Sigma^{\parallel})/(\Sigma^{\perp} + \Sigma^{\parallel})$.

It is seen that $Re(n^{\perp} - n^{\parallel})$ has maxima where $dE/d\omega$ is maximum[6].

It is a suggestive possibility that the effects discussed here and in A and B will allow the introduction of an new family of "optical" instruments for the handling of very high energy photon beams.

We are grateful to the "Gruppo Calcoli Numerici" of the Frascati Laboratories for the assistance given to us in the use of the IBM 1620 competer.

References

(1) - N. Cabibbo, G. Da Prato, G. De Franceschi, U. Mosco: Phys. Rev. Lett. 9, 270 (1962) and N. Cabibbo, G. Da Prato, G. De Franceschi, U. Mosco: Absorption of gamma rays in crystals and the production and analysis of linearly polarized gamma rays. - LNF-62/70, to be published in the 'Nuovo Cimento", here after referred to as A and B.

(2) - The diagonality is due to our choice of the basic vectors and the symmetry of our situation in respect to the (001) plane; the exponential form can be assumed because the coherent action of the crystal is limited to microscopic dimensions because of multiple scattering of the produced electrons.

(3) - We use the usual units with $\hbar = c = 1$ and also $m_e = 1$.

(4) - The n's introduced here are essentially the amplitudes for forward Delbruck scattering in a crystal.

(5) - This is obtained from eqs. 15') and A10) in B neglecting q^2 in respect to m_e and B, an approximation which was found very accurate and greatly simplifies the integration.

(6) - In these points $\Sigma^{\perp} - \Sigma^{\parallel}$ is also maximum, since $\Sigma^{\perp} + \Sigma^{\parallel}$ is dominated by the incoherent contribution and relatively flat.

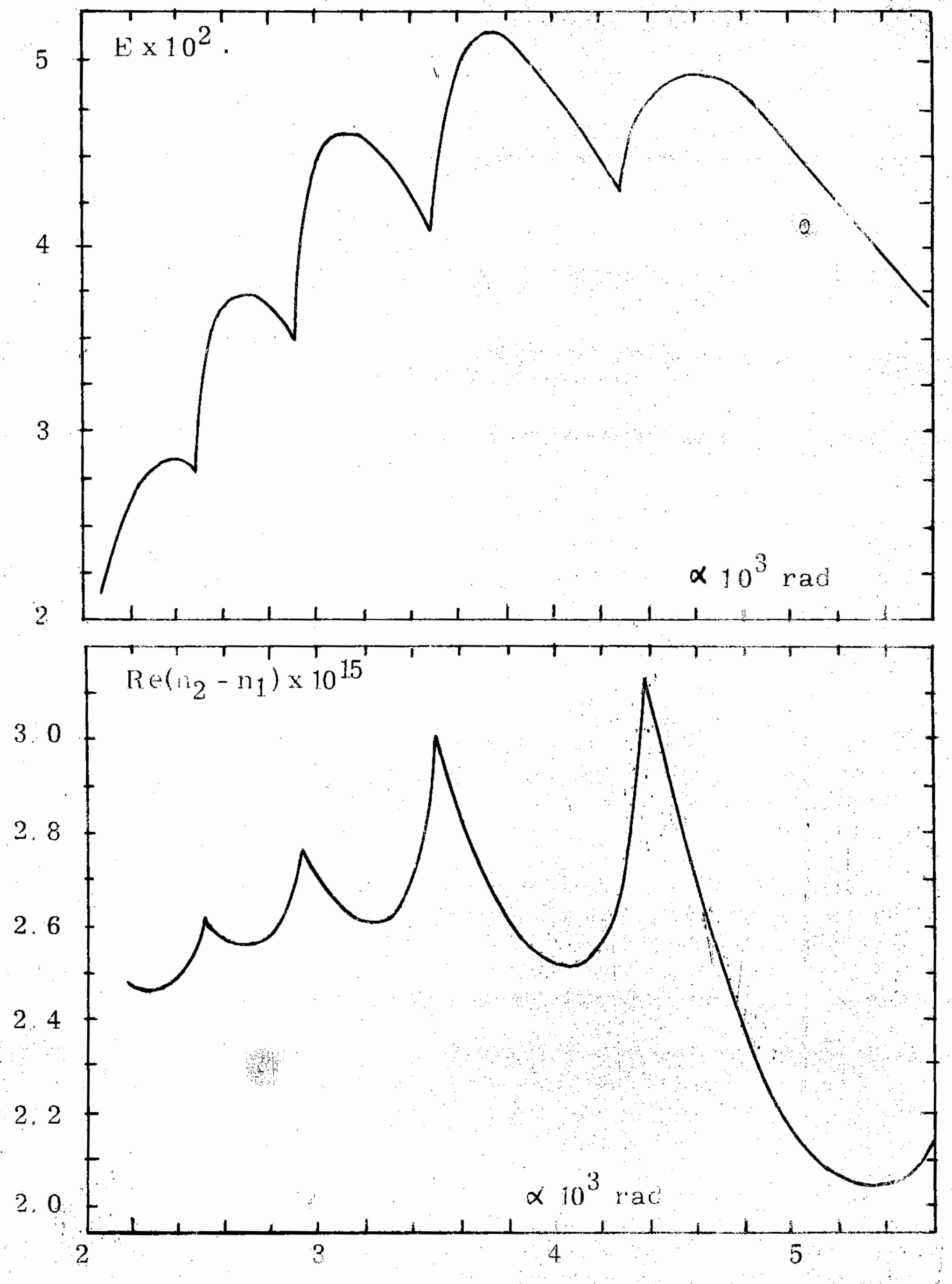

FIG. 1 - E (upper part) and Re(n' - n") (lower part) as a function of α at ω = 6 GeV

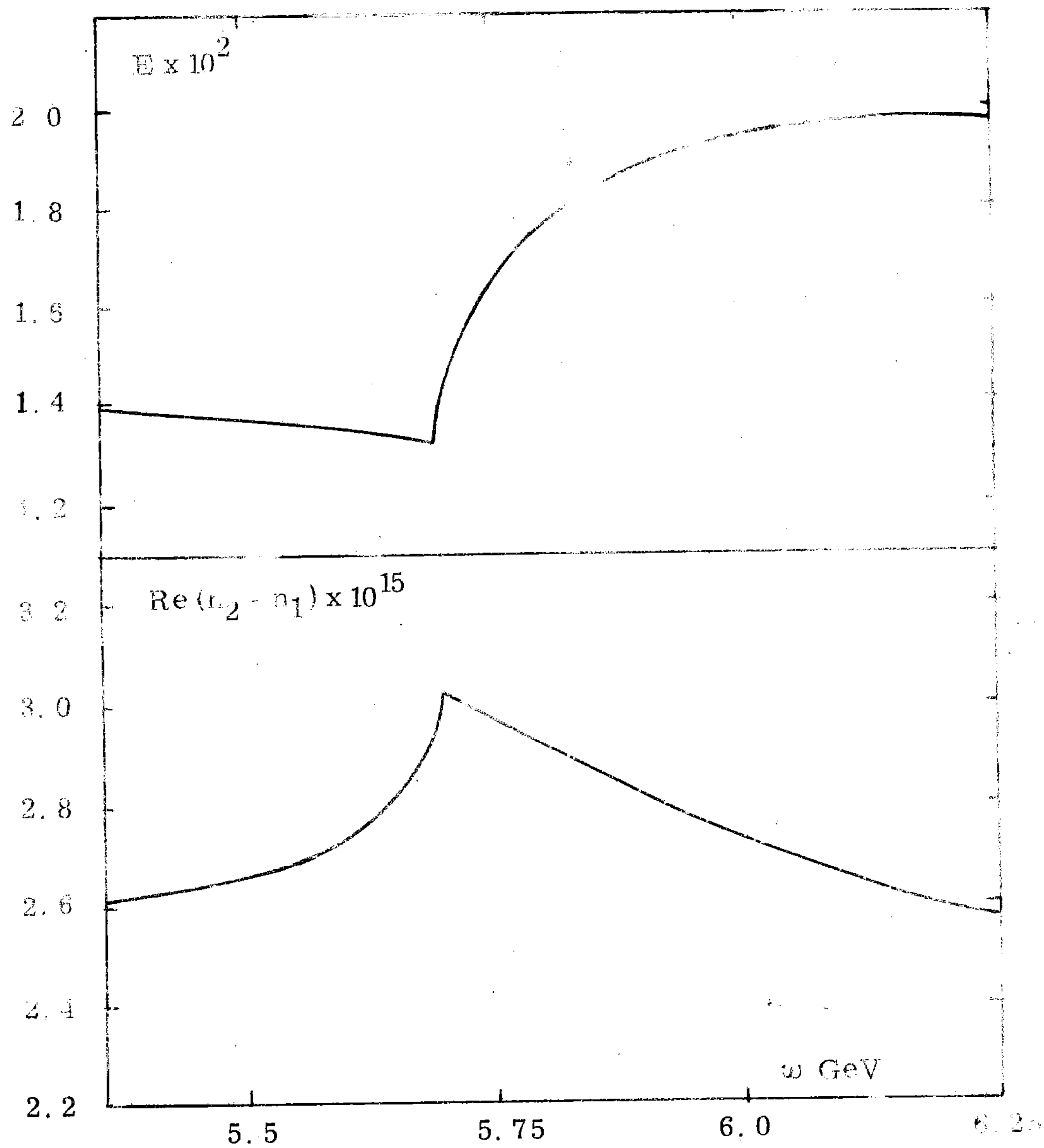

FIG. 2 - E (upper part) and Re ($n^{\perp}$ - $n^{\parallel}$) (lower part) as a function of ω at δ = 3.7 mrad.